



### 332907 Sensor Maintenance Optimization Based On a Prognostic Model

**Monday, November 4, 2013**

Grand Ballroom B (Hilton)

**Kris Villez**, Process Engineering, Eawag, Dübendorf, Switzerland

#### Introduction

Sensors based on an Ion-Selective Electrode (ISE) are characterized by an intercept ( $\alpha$ ) and slope ( $\beta$ ) which link a transmitter output voltage (or electrical current) to the variable of interest. In this work, we write:

$$y = \alpha + \beta \cdot z$$

with  $z$  the variable of interest and  $y$  the measured voltage.

It is typical that the sensor parameters ( $\alpha$  and  $\beta$ ) change over time, both due to reversible (e.g., fouling) and irreversible (e.g., buffer leaking) effects. It is therefore recommended to recalibrate such sensors on a regular basis [1-3]. Unfortunately, this can be a time-consuming task as limited efforts have been made to optimize the effort in maintenance and calibration of ISE sensors [4]. In this work, a state-space model is formulated which models the evolution of a sensor's parameters as a continuous time-varying process. Given this process model, reference measurements are used to update the parameter estimates by means of the (linear) Kalman filter. The resulting parameter estimates and the associated variance-covariance matrices allow prediction of future parameter estimates and covariances. Based on these predictions, future reference measurements can be scheduled as soon as the expected confidence intervals are considered too large. Methodology Simulated sensor

A pH sensor is simulated as follows. The intercept ( $\alpha$ ) and slope ( $\beta$ ) are joined in a single state vector,  $x$ :

$$x = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (1)$$

Given this definition, the evolution in time of this vector is given in the form of a continuous-time process:

$$\dot{x}(t) = A \cdot x(t) + B \cdot u(t) + \dot{v}(t) \quad (2)$$

with  $u(t)$  an external input,  $A$  and  $B$  defining the linear, deterministic parts of the model and  $v$  representing the stochastic input disturbances. This is a general representation of a Wiener process.

The sensor's parameters are however considered to evolve independently of their own values and independent of any deterministic input so that one can simplify this model as follows:

$$\dot{x}(t) = \dot{v}(t) = \frac{dv}{dt} \quad (3)$$

The resulting model is a special case of the general Wiener process, namely that of a multivariate Brownian motion. The input disturbances are considered to be zero mean and to have a diagonal covariance matrix, meaning that intercept and slope evolve independently from each other. More concretely, one writes:

$$E[\dot{dv}(t) \cdot \dot{dv}(t)^T] = Q = q \cdot I \quad (4)$$

$$q = 0.0001$$

The actual values for  $x$  are of course not available and can only indirectly be estimated from the measured voltages. The value for the voltage measurement depends on the pH ( $z$ ), the sensor parameters ( $x(t)$ ) and a measurement error (white noise):

$$y_z(t) = \alpha(t) + \beta(t) \cdot z = [1 \ z] \cdot \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix} = c_z \cdot x(t) + w_z(t) \quad (5)$$

It is assumed that the considered measurements are taken with a sensor placed in a buffer solution with known pH value and following steady-state. More concretely, pH buffer solutions are considered available at 4, 7 and 10, and measurements are taken for each buffer solution in a very short interval. These intervals are considered sufficiently small so that one can consider the measurements as simultaneous. In this case, the more specific measurement equation is the following:

$$y(t) = \begin{bmatrix} y_{10}(t) \\ y_7(t) \\ y_4(t) \end{bmatrix} + \begin{bmatrix} w_{10}(t) \\ w_7(t) \\ w_4(t) \end{bmatrix} = \alpha(t) + \beta(t) \cdot \begin{bmatrix} 10 \\ 7 \\ 4 \end{bmatrix} + w(t) = \begin{bmatrix} 1 & 10 \\ 1 & 7 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix} = C \cdot x(t) + w(t) \quad (6)$$

The resulting vector  $y(t)$  contains the noisy voltage measurements at buffer values 10, 7 and 4. These are further referred to as reference measurements. The vector  $w$  is a white noise vector, i.e., zero-mean and with a diagonal variance-covariance matrix:

$$E\{[w(t)] \cdot [w(t)]^T\} = R = r \cdot I \quad (7)$$

$$r = 0.01$$

The initial values for alpha and beta are set to 4 (intercept 4V at pH=0) and 16/14=1.14 (slope of 16V units over 14 pH units). This completely specifies the executed sensor model simulations. Filtering

The Kalman filter [5] is used to obtain on-line estimates of the sensor parameters. To this end, prediction equations are derived by integration of Eq. 3 from a previous time instant,  $t(k-1)$ , to the current,  $t(k)$  [Harvey]:

$$\mu_x(k+1|k) = \mu_x(k|k)$$

$$\Sigma_x(k+1|k) = \Sigma_x(k|k) + \delta(k+1) \cdot Q \quad (8)$$

$$\delta(k+1) = t(k+1) - t(k)$$

where  $\mu_x(k|k)$  and  $\Sigma_x(k|k)$  are the current state estimate and associated covariance matrix and  $\mu_x(k+1|k)$  and  $\Sigma_x(k+1|k)$  are the predicted (future) state estimate and associated covariance matrix. The above equations signify that the expected future state vector is equal to the current state estimate and the covariance matrix of the estimates increases linearly with time.

When reference measurements,  $y(k+1)$ , are present at a given time,  $t(k+1)$ , they are used to update the state estimates. For this, the so called update equations are:

$$\mu_y(k+1|k) = C \cdot \mu_x(k+1|k)$$

$$\Sigma_y(k+1|k) = C \cdot \Sigma_x(k+1|k) \cdot C^T + R$$

$$K(k+1) = \Sigma_x(k+1|k) \cdot C^T \cdot [\Sigma_y(k+1|k)]^{-1} \quad (9)$$

$$\mu_x(k+1|k+1) = \mu_x(k+1|k) + K(k+1) \cdot (y(k+1) - \mu_y(k+1|k))$$

$$\Sigma_x(k+1|k+1) = [I - K(k+1) \cdot C] \cdot \Sigma_x(k+1|k)$$

The update equations are executed whenever a set of measurements for the buffers is available. In contrast, the prediction equations can be executed at any time. Prediction

At any point in time, the above prediction equations can be used to estimate the future values for the sensor parameters and associated covariance matrix. In addition, one can also estimate the expected voltage value and associated variance for any pH value of interest ( $z$ ). To this end, one writes:

$$\mu_z(k+1|k) = C_z \cdot \mu_x(k+1|k)$$

$$\sigma_z(k+1|k) = C_z \cdot \Sigma_x(k+1|k) \cdot C_z^T + r \quad (10)$$

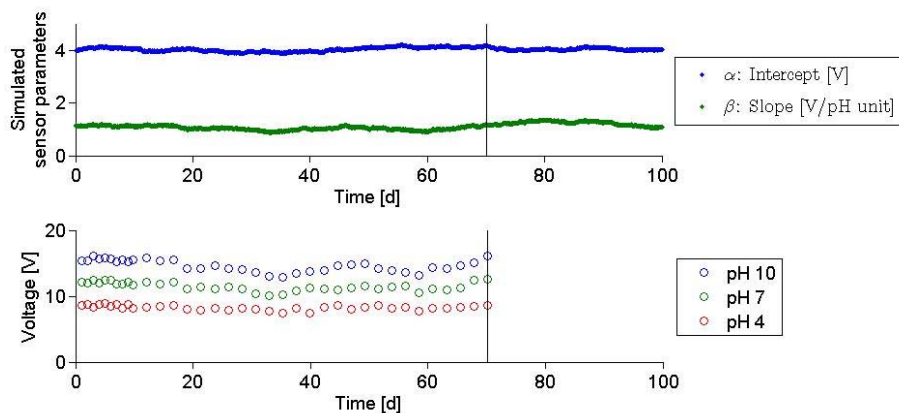
Scheduling of reference measurements Default scheduling

$$C_z = \begin{bmatrix} 1 & z \end{bmatrix}$$

In a first simulation study, reference measurements are taken at predefined time intervals, namely on a weekly repeated schedule in which measurements are taken on the first (1), second (2), third (3) and fifth (5) day. This leads to time intervals of 1, 2 and 3 days between consecutive reference measurements. This is continued for 10 weeks (70 days). After this time, the sensors behaviour is simulated for another 4 weeks (until day 98). Optimized scheduling

In a second simulation study, the lowest predicted variances for the measurements observed for pH values 5 and 8 ( $\Sigma_{y,5}$ ,  $\Sigma_{y,8}$ ) in the first simulation study are used as a reference. A new measurement is taken for the buffer solutions (pH = 4, 7 10) as soon as the predicted variances are higher than 1.1 times these reference values. Results

Due to space limitation, only results for the optimized scheduling are shown here. As discussed above, reference measurements are taken only when the variance of the expected voltage at pH 5 or pH 8 is 1.1 times higher than the minimal value obtained in the first simulation. Figure 1 shows the simulated parameters as well as the reference measurements taken with this schedule. In this case, the total number of reference measurements is 36 while in the default scheduling simulation (default scheduling) it is 40 (not shown).



**Figure 1: Optimized scheduling - Simulated data. The vertical line indicates the end of the measurement collection. Top: Simulated sensor parameters. Bottom: Reference voltage measurements for three buffer solutions. The frequency of measurements is relatively high at the beginning and is reduced and constant as from the 10<sup>th</sup> day until the last reference measurement (day 70).**

Figure 2 and 3 display the Kalman filtering results. These are similar to the default scheduling simulation except that the variances in the parameter estimates are reduced faster at the beginning of the simulated period. This is because the optimized scheduling results in a higher sampling frequency at the beginning of the simulated period. After the parameter variances are below the set threshold, this frequency is reduced.

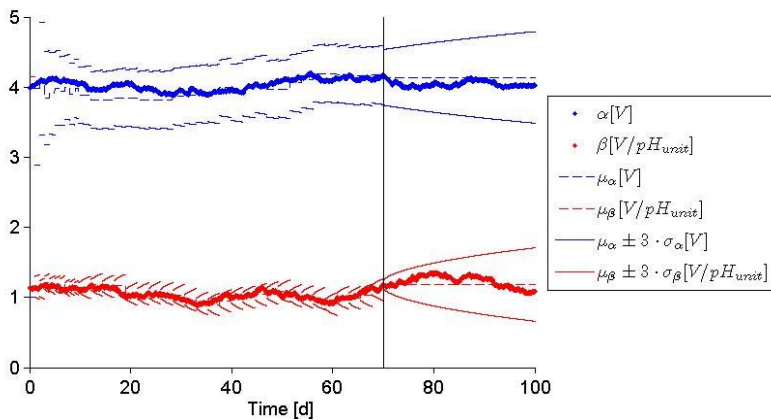


Figure 2: Optimized scheduling - True

sensor parameters (.), estimates (--) and 3-sigma confidence intervals (-). The vertical line indicates the end of the measurement collection. The parameter estimates remain constant while the confidence intervals widen after this time.

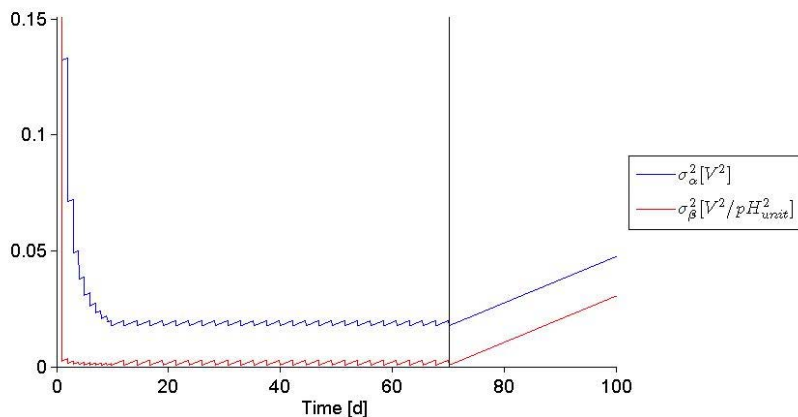


Figure 3: Optimized scheduling - Variances

of the parameter estimates. The vertical line indicates the end of the measurement collection. The uncertainty about the slope (beta) is higher than the uncertainty about the intercept (alpha). As in the default scheduling case, the variances are reduced at each reference measurement and increase linearly over time between reference measurements. The reduction in variance at the beginning of the simulated period is faster however compared to the default scheduling case (see Figure 3). Discussion

In this work, a prognostic state-space model is proposed to model the time-varying nature of a typical ISE sensor. By means of such model, reference measurements can be used to estimate the values of the sensor's characterizing parameters as well as the predict the future uncertainty in its measurements. Based on such uncertainty evaluation, it is possible to schedule new reference measurements so to maintain uncertainty (variance) below a pre-set level. This approach is tested in a simulated study which suggests that a reduction of maintenance effort can effectively be bargained against accuracy. Furthermore, the proposed scheduling effectively results in higher maintenance efforts when the variances of parameter estimates are higher. References

- [1] Rieger, L., Alex, J., Winkler, S., Moehler, M., Thomann, M., & Siegrist, H. (2003). Progress in sensor technology- progress in process control? Part I: Sensor property investigation and classification. *Water Science & Technology*, 47(2), 103-112.
- [2] Rieger, L., Thomann, M., Gujer, W., & Siegrist, H. (2005). Quantifying the uncertainty of on-line sensors at WWTPs during field operation. *Water research*, 39(20), 5162-5174.
- [3] Nopens, I.; Villez, K.; Rieger, L.; Vanrolleghem, P.A. (2007). Monitoring the water cycle - state of the art and future needs. *IWA Yearbook*, 2007, 33-36.
- [4] Rieger, L., & Vanrolleghem, P. A. (2008). monEAU: a platform for water quality monitoring networks. *Water Science and Technology*, 57(7), 1079-1086.
- [5] Brown, R. G., & Hwang, P. Y. (1992). *Introduction to random signals and applied Kalman filtering* (Vol. 1, No. 1). New York: John Wiley & Sons.
- [6] Haykin, S. S. (Ed.). (2001). *Kalman filtering and neural networks*. New York: Wiley.

Extended Abstract: File Not Uploaded

See more of this Session: [Poster Session in Information Management and Intelligent Systems](#)  
See more of this Group/Topical: [Computing and Systems Technology Division](#)