Exercise 2

Modelling Aquatic Ecosystems FS26

Today’s plan

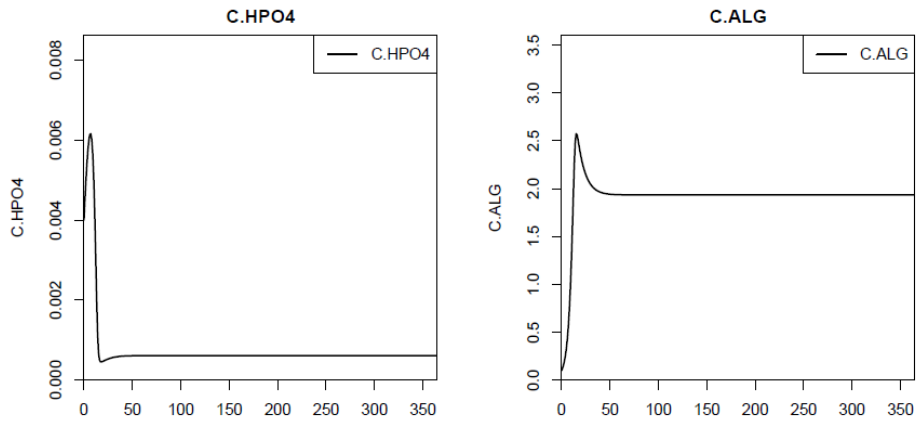
- Q&A of last week’s exercise
- Intro to phytoplankton – zooplankton model
- Recap on elements in process rates
- Break
- Work on the exercise on your own
- Discussion of theory questions

Q&A of last week's exercise

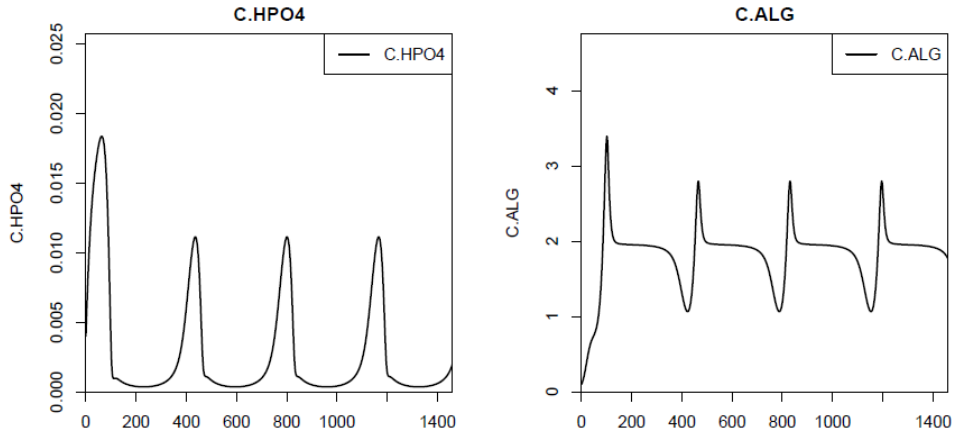
Are there any open questions on Exercise 1?

Homework solution visualization:

constant external conditions:



seasonally varying external conditions:



Reminder: The solutions are available on the course website
https://www.eawag.ch/en/department/siam/teaching/modelling-aquatic-ecosystems/#c21285

Question 1

How can you derive the total (net) transformation rate of $C_{\mathrm{HPO_4^{-2}}}$ and C_{ALG} from the process table (Table 11.1) and the process rates (Table 11.2)?

Hint: see equation (4.1) in the manuscript. What are the units?

Question 1 - Answer

$\alpha_{P,ALG}$ units of phosphate are consumed to produce one unit of algae

Process Table

Process i	Substances / Organisms j: HPO4 [gP/m³]	ALG [gDM/m³]	Rate ρ
Growth of algae	$-\alpha_{P,ALG}$	1	$\rho_{gro,ALG} = k_{gro,ALG} \frac{C_{HPO_4^{2-}}}{K_{HPO_4^{2-},ALG} + C_{HPO_4^{2-}}} C_{ALG}$
Death of algae		-1	$\rho_{death,ALG} = k_{death,ALG} C_{ALG}$

growth rate modified by limitation term in phosphate
(phosphate is consumed, rate has to go slower if phosphate declines)

linear death rate

one unit of algae disappears from the modelled part of the system
(no mass conservation!)

$$r_j = \sum_{i=1}^{n_P} \nu_{ij} \rho_i$$

r = Net transformation rate

ν = Substance-specific stoichiometric coefficients

ρ = Process rate

j = Substance

i = Process

Question 1 - Answer

$$r_{Alg} = k_{gro,\,Alg} \times \frac{C_P}{K + C_P} \times C_{Alg} - k_{death,\,Alg} \times C_{Alg}$$

$$[r_{Alg}] = \frac{1}{d} \times \frac{gDM}{m^3} = \frac{gDM}{d \times m^3}$$

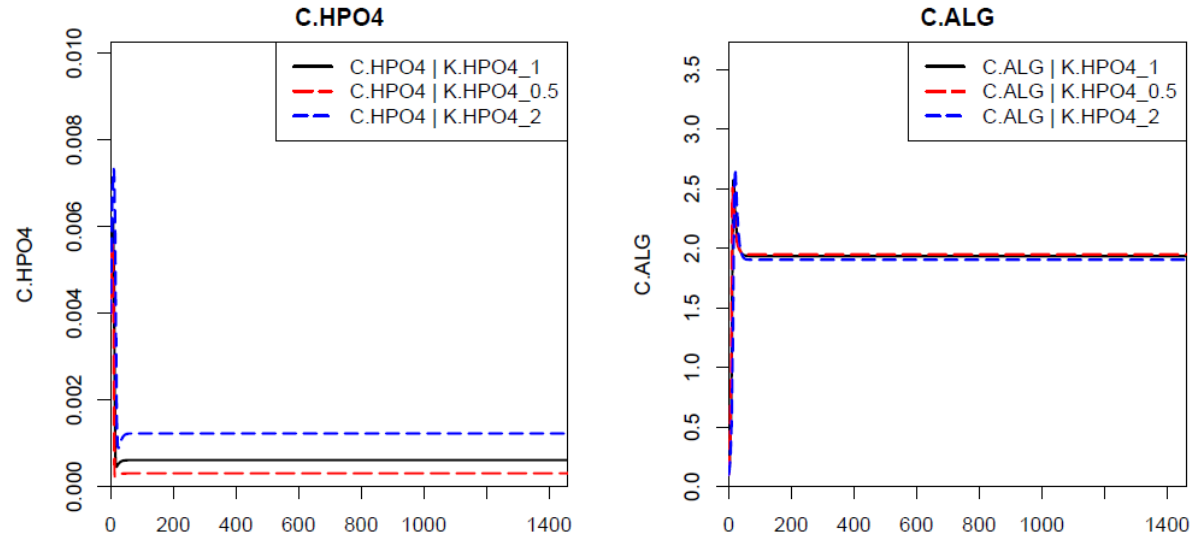
$$r_P = -\alpha_{P,Alg} \times k_{gro,\,Alg} \times \frac{C_P}{K + C_P} \times C_{Alg}$$

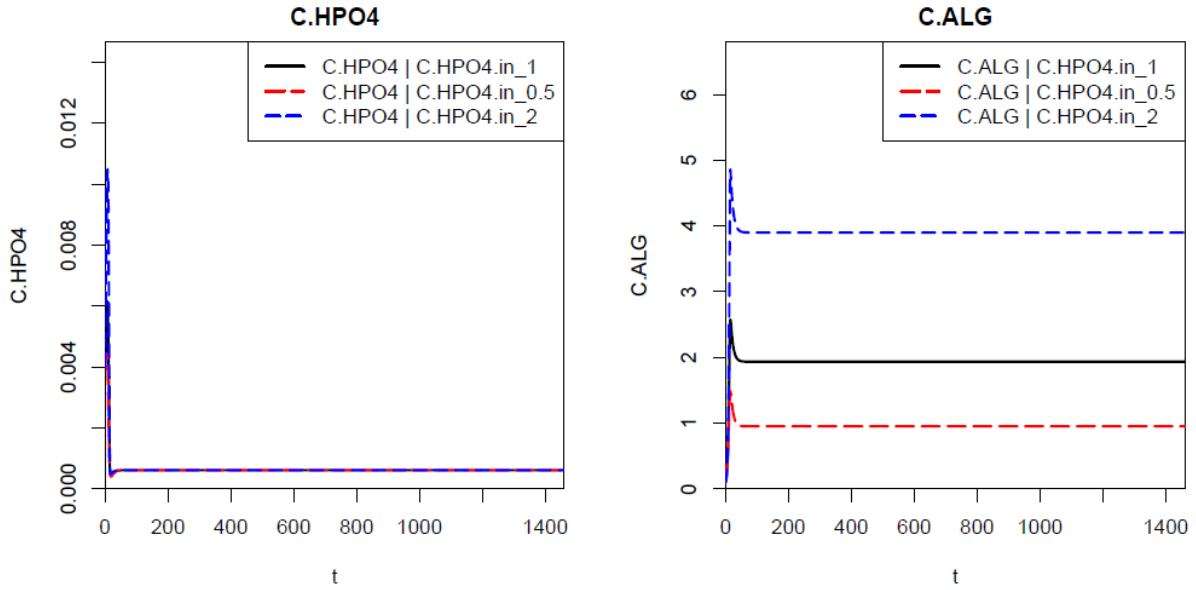
$$[r_P] = \frac{gP}{gDM} \times \frac{1}{d} \times \frac{gDM}{m^3} = \frac{gP}{d \times m^3}$$

Question 2

Look at the state variables $C_{\mathrm{HPO_4^{2-}}}$ and C_{ALG}. Which of them is more sensitive to the parameter $K_{\mathrm{HPO_4^{2-},\ ALG}}$ and which of them is more sensitive to $C_{\mathrm{in,\ HPO_4^{2-}}}$? Do you understand why?

Question 2 - Visualization





Look at the Manuscript in the chapter 11.1 (pp.170-174)

Intro to exercise 2
phytoplankton – zooplankton model

$\alpha_{P,ALG}$ units of phosphate are consumed to produce one unit of algae (units indicated above)

one unit of algae disappears from the modelled part of the system (no mass conservation!)

Process	Substances / Organisms			Rate
	HPO_4^{2-}	ALG	ZOO	
	gP	gDM	gDM	
Growth of algae	$-\alpha_{P,ALG}$	1		$\rho_{gro,ALG}$
Death of algae		-1		$\rho_{death,ALG}$
Growth of zooplankton		$-\frac{1}{Y_{ZOO}}$	1	$\rho_{gro,ZOO}$
Death of zooplankton			-1	$\rho_{death,ZOO}$

process rates **???**

$1/Y_{ZOO}$ (> 1) units of algae are consumed to produce one unit of zooplankton (no mass conservation!)

one unit of zooplanktion disappears from the modelled part of the system (no mass conservation!)

What constitute process rates ?

Process Rates (4.2)

Process rates depend on nutrients and organisms concentrations and can often be written in the form of a base rate with limiting and inhibiting factors.

Process formulation with multiplicative factors:

rate parameter

temperature dependence

light dependence

$$\rho_{\mathrm{gro,ALG,NH_4^+}} = k_{\mathrm{gro,ALG},T_0} \cdot f_{\mathrm{temp}}(T) \cdot f_{\mathrm{rad}}(I) \cdot f_{\mathrm{lim}}(C_{\mathrm{HPO_4^{2-}}}, C_{\mathrm{NH_4^+}}, C_{\mathrm{NO_3^-}}) \cdot C_{\mathrm{ALG}}$$

nutrient limitation

rate linear in algae concentration

Process Rates (11.2)

Model with constant driving forces (Table 11.5):

phosphate limitation of algae growth

Rate	Rate expression
$\rho_{\text{gro,ALG}}$	$k_{\text{gro,ALG}} \dfrac{C_{\text{HPO}_4^{2-}}}{K_{\text{HPO}_4^{2-},\text{ALG}} + C_{\text{HPO}_4^{2-}}} C_{\text{ALG}}$
$\rho_{\text{death,ALG}}$	$k_{\text{death,ALG}}\, C_{\text{ALG}}$
$\rho_{\text{gro,ZOO}}$	$k_{\text{gro,ZOO}}\, C_{\text{ALG}}\, C_{\text{ZOO}}$
$\rho_{\text{death,ZOO}}$	$k_{\text{death,ZOO}}\, C_{\text{ZOO}}$

zooplankton growth rate linear in zooplankton with a linear limitation in algae

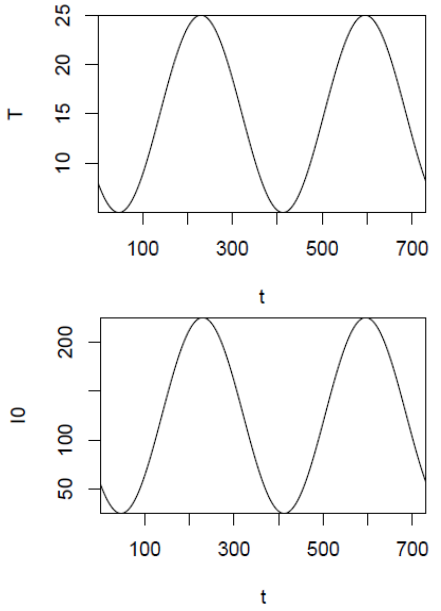
linear death rate

Process Rates (11.2)

Model with seasonally varying driving forces (Table 11.6):

$$T(t) = \frac{T_{\max} + T_{\min}}{2} + \frac{T_{\max} - T_{\min}}{2} \cos\left(2\pi \frac{t - t_{\max}}{t_{\text{per}}}\right)$$

$$I_0(t) = \frac{I_{0,\max} + I_{0,\min}}{2} + \frac{I_{0,\max} - I_{0,\min}}{2} \cos\left(2\pi \frac{t - t_{\max}}{t_{\text{per}}}\right)$$



Rate	Rate expression
$\rho_{\text{gro,ALG}}$	$k_{\text{gro,ALG},T_0} \cdot \exp\left(\beta_{\text{ALG}}(T - T_0)\right) \cdot \frac{1}{\lambda h} \log\left(\frac{K_I + I_0}{K_I + I_0 \exp(-\lambda h)}\right) \cdot \frac{C_{\text{HPO}_4^{2-}}}{K_{\text{HPO}_4^{2-},\text{ALG}} + C_{\text{HPO}_4^{2-}}} \cdot C_{\text{ALG}}$
$\rho_{\text{death,ALG}}$	$k_{\text{death,ALG}}\, C_{\text{ALG}}$
$\rho_{\text{gro,ZOO}}$	$k_{\text{gro,ZOO},T_0} \cdot \exp\left(\beta_{\text{ZOO}}(T - T_0)\right) \cdot C_{\text{ALG}}\, C_{\text{ZOO}}$
$\rho_{\text{death,ZOO}}$	$k_{\text{death,ZOO}}\, C_{\text{ZOO}}$

temperature dependence

light dependence

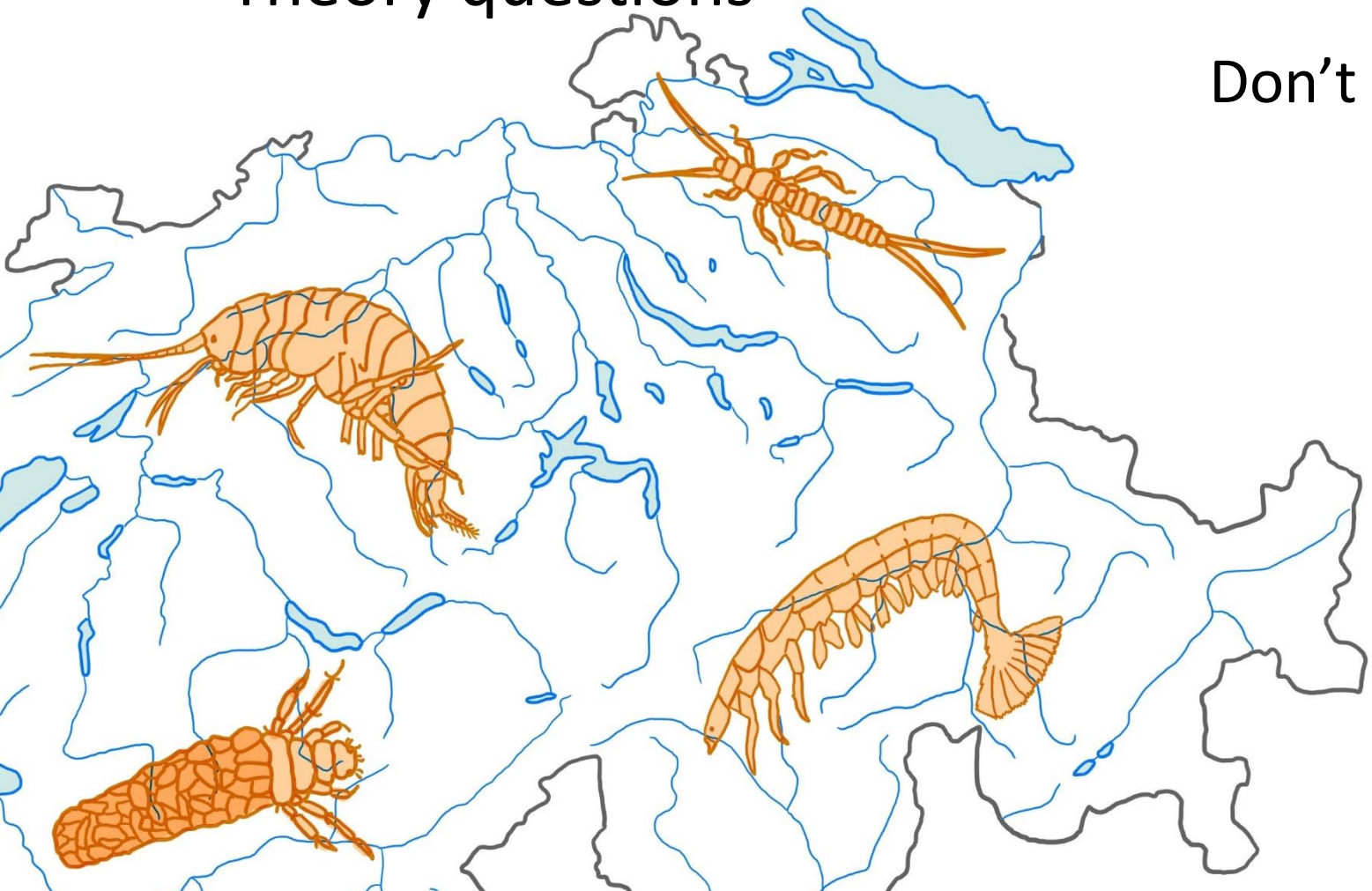
Time to work on Exercise 2

Theory questions

- Are the algae concentrations controlled bottom-up (by phosphate limitation) or top-down (by grazing of zooplankton)?
- What is the reason for oscillating concentrations under constant driving forces? What happens when you introduce periodic driving forces?
- What are the main deficits of the model compared to a real lake?
- What is your expectation regarding the response of the model to the change in each parameter, does the result match your expectation and can you explain the observed changes?

Homeworks:

- Task 4 – Sensitivity analysis
- Theory questions



Don’t hesitate to send us an e-mail if you have any questions.

Have a nice day !