

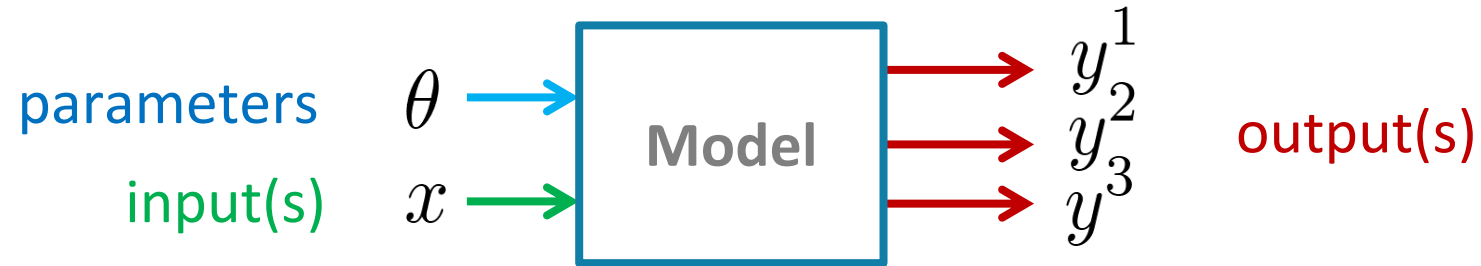
# Exercise 6

Modelling Aquatic Ecosystems FS24

# Today's agenda

- Work on task 1-2, discuss the questions and results
- Break
- Work on task 3-4, discuss the questions and results
- Open questions

# Task 1 - Uncertainty analysis



I know the *uncertainty of the inputs or parameters.*

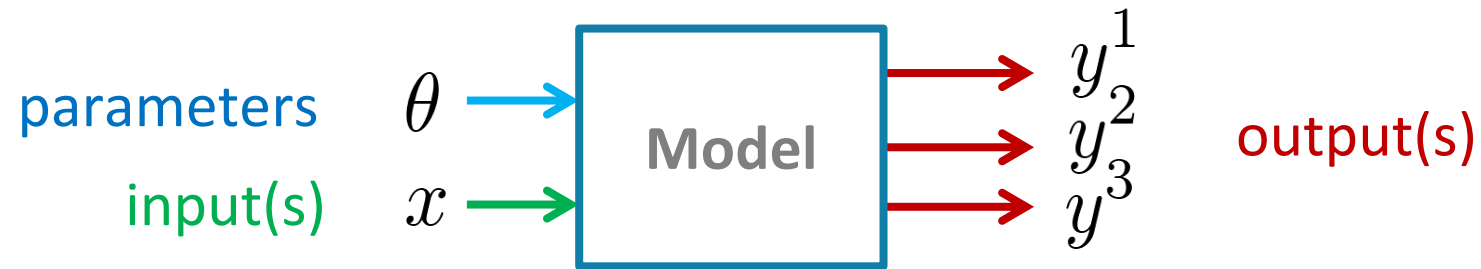
What is the resulting uncertainty of the *outputs*?

How can I compute it? E.g., **Monte Carlo Error propagation**

# Task 1 - Questions

- How would you decide on the standard deviation for the different parameters?
- How large has  $N$  to be to get stable results?
- (optional) Compute the mean and sd of the outputs at  $t = 365$
- (optional) Make a histogram of the model outputs at  $t = 365$

# Task 2-3-4 – Parameter estimation



What *parameters* make the *outputs* most similar to the observations?

Where do we have information about parameter values?

- I. Laboratory experiments
- II. Scientific literature
- III. Calibration with observational data**

What are the different model calibration techniques?

- i. Manual calibration
- ii. Minimizing a loss function
- iii. Maximum Likelihood estimation – a special loss function**
- iv. Bayesian inference – combine field data with other information**

## Task 2 – Likelihood function

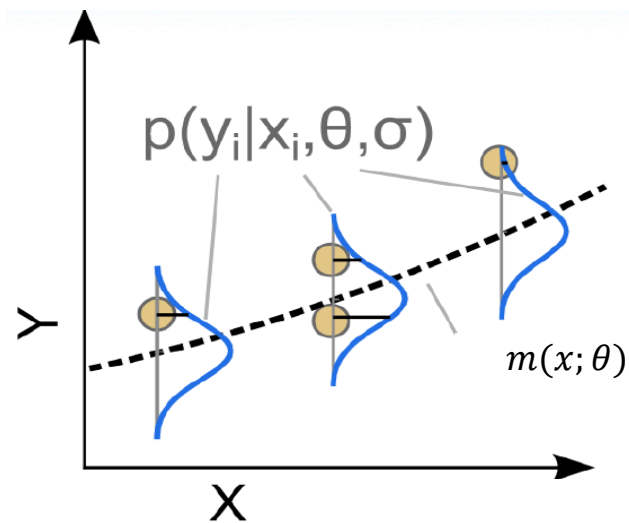
A likelihood function  $p(\text{data}|\theta)$  answers the following question:

“Given a *stochastic* model that generates random data. If the parameters are set to  $\theta$ , what is probability (density) that the randomly generated data equal the observed?”

# Task 2 – Likelihood function

deterministic model

$$Y_i = \overbrace{m(x; \theta)} + \underbrace{\epsilon_i}_{\text{noise}}$$



Likelihood for a single observations

$$p_i(y_i | x_i, \theta, \sigma) = \Phi \left( \frac{y_i - m(x; \theta)}{\sigma} \right)$$

Likelihood for all observations

$$p(\mathbf{y} | \mathbf{x}, \theta, \sigma) = \prod_i p_i(y_i | x_i, \theta, \sigma)$$

Time to work on the exercise



## Task 3 - Questions

- What happens if you choose different initial values `par.ini`?
- How do you interpret `sd.obs.HPO4` and `sd.obs.ALG`?

# Task 4 - Questions

- What can you get out of the parameter histograms?
- Experiment with different prior distributions.
- Try different initial values. What happens, if the initial values are very far off?
- What happens if the sample size is too small?
- Are some parameters correlated?

Open questions?